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Reg No.: $\qquad$ Name: $\qquad$
APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

## Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS
Max. Marks: 100
Duration: 3 Hours

## PART A

Answer any two full questions, each carries 15 marks.
1 a) Find the points where Cauchy-Riemann equations are satisfied for the function $f(z)=x y^{2}+i x^{2} y$. Where does $f^{\prime}(z)$ exist? Is the function $f(z)$ analytic at those points?
b) If $v=e^{x}(x \sin y+y \cos y)$, find an analytic function $f(z)=u+i v$.

2 a) Show that $\mathrm{u}=\mathrm{x}^{2}-\mathrm{y}^{2}-\mathrm{y}$ is harmonic. Also find the corresponding conjugate harmonic function.
b) (i) Find a bilinear transformation which maps ( $-i, 0$, i) onto $(0,-1, \infty)$.
(ii) Test the continuity at $\mathrm{z}=0$, if $\mathrm{f}(\mathrm{z})=\frac{I m z}{|z|}, z \neq 0$

$$
=0, z=0
$$

3 a) Find the image of the lines $\mathrm{x}=1, \mathrm{y}=2$ and $\mathrm{x}>0, \mathrm{y}<0$ under the mapping $\mathrm{W}=\mathrm{z}^{2}$
b) Find the image of the semi-infinite strip $x>0,0<y<2$ under the transformation $\mathrm{w}=\mathrm{iz}+1$. Draw the regions.

## PART B

Answer any two full questions, each carries 15 marks.
4 a) Evaluate $\oint R e z^{2} d z$ over the boundary $C$ of the square with vertices $0, i, 1+i, 1$ clockwise
b) Evaluate $\int \frac{4-3 z}{z(z-1)}$ dz over the circle $|z|=\frac{3}{2}$
c) Evaluate $\int \frac{3 z^{2}+7 z+1}{z+1}$ dz over the circle $|z+i|=1$

5 a) Expand $\frac{z}{(z-1)(z-2)}$ in (1) $0<|z-2|<1$, (2) $|z-1|>1$
b) Evaluate $\int_{0}^{2 \pi} \frac{1}{2+\cos \theta} \mathrm{d} \theta$

6 a) Using Residue theorem evaluate $\int \frac{z^{2}}{(z-1)^{2}(z+2)}$ dz over the circle $|z|=3$
b) Find the Taylor series of $\frac{\sin z}{z-\pi}$ about the point $\mathrm{z}=\pi$
c) Evaluate $\int \frac{\sin z}{z^{6}}$ dz over the circle $|z|=2$ using Cauchy's Residue theorem.

PART C
Answer any two full questions, each carries 20 marks.
7 a) Solve by Gauss-Elimination method $x+y+z=6, x+2 y-3 z=-4,-x-4 y+9 z=18$.
b) Find the values of ' $a$ ' and ' $b$ ' for which the system of equations $x+y+2 z=2$, $2 x-y+3 z=10,5 x-y+a z=b$ has:
(i) no solution
(ii) unique solution
(iii) infinite number of solutions.
c) Verify whether the vectors $(1,2,1,2),(3,1,-2,1),(4,-3,-1,3)$ and $(2,4,2,4)$ are linearly independent in $\mathrm{R}^{4}$.

8 a) Write down the matrix associated with the quadratic form $8 x_{1}{ }^{2}+7 x_{2}{ }^{2}+3 x_{3}{ }^{2}-12 x_{1} x_{2}$ $-8 x_{2} x_{3}+4 x_{3} x_{1}$. By finding eigen values, determine nature of the quadratic form.
b) Diagonalise the matrix $\mathrm{A}=\left[\begin{array}{ccc}1 & -2 & 0 \\ -2 & 0 & 2 \\ 0 & 2 & -1\end{array}\right]$
c) If A is a symmetric matrix, verify whether $\mathrm{AA}^{\mathrm{T}}$ and $\mathrm{A}^{\mathrm{T}} \mathrm{A}$ are symmetric?

9 a) $\quad$ Find the eigen vectors of $A=\left[\begin{array}{lll}3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1\end{array}\right]$
b)

Find the null space of $A X=0$ if $A=\left[\begin{array}{cccc}1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \\ 4 & 4 & -1 & 9\end{array}\right]$
c) Verify whether $A=\left[\begin{array}{ccc}1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta\end{array}\right] \quad$ is orthogonal.

What can you say about determinant of an orthogonal matrix? Prove or disprove the result.

