$\qquad$ Name:

# APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY 

 FIRST SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018
## Course Code: MA101

Course Name: CALCULUS
Max. Marks: 100
Duration: 3 Hours

## PART A <br> Answer all questions, each carries 5 marks.

1 a) Determine whether the series $\sum_{k=0}^{\infty} \frac{5}{4^{k}}$ converges. If so, find the sum
b) Examine the convergence of $\sum\left(\frac{k}{k+1}\right)^{k^{2}}$

2 a) Find the slope of the surface $z=x e^{-y}+5 y$ in the $y$ direction at the point $(4,0)$
b) Show the function $f(x, y)=e^{x} \sin y+e^{y} \cos x$ satisfies the Laplace's equation

$$
\begin{equation*}
f_{x x}+f_{y y}=0 \tag{2}
\end{equation*}
$$

a) Find the directional derivative of $f(x, y, z)=x^{3} z-y x^{2}+z^{2}$ at $\mathrm{P}(2,-1,1)$ in the direction of $3 \vec{\imath}-\boldsymbol{j}+2 \mathbf{k}$
b) Find the unit tangent vector and unit normal vector to the curve

$$
\begin{equation*}
\boldsymbol{r}(t)=4 \cos t \boldsymbol{i}+4 \sin t \boldsymbol{j}+t \boldsymbol{k} \text { at } t=\frac{\pi}{2} \tag{3}
\end{equation*}
$$

a) Using double integration, evaluate the area enclosed by the lines

$$
\begin{equation*}
x=0, \quad y=0, \frac{x}{a}+\frac{y}{b}=1 \tag{2}
\end{equation*}
$$

b)

Evaluate $\quad \int_{-1}^{2} \int_{0}^{2} \int_{0}^{1}\left(x^{2}+y^{2}+z^{2}\right) d x d y d z$
a) If $\boldsymbol{F}(x, y, z)=x^{2} \boldsymbol{i}-3 \boldsymbol{j}+y z^{2} \boldsymbol{k}$ find $\operatorname{div} \boldsymbol{F}$
b) Find the work done by the force field $\boldsymbol{F}=x y \boldsymbol{i}+y z \boldsymbol{j}+z x \boldsymbol{k}$ on a particle that
moves along the curve C: $x=t, y=t^{2}, z=t^{3}, 0 \leq t \leq 1$
a) Use Green's theorem to evaluate $\int_{c}(x d y-y d x)$, where $c$ is the circle $x^{2}+y^{2}=$ $a^{2}$
b) If S is any closed surface enclosing a volume V and $\boldsymbol{F}=x \boldsymbol{i}+2 y \boldsymbol{j}+3 z \boldsymbol{k}$ show that $\iint_{S} F . \mathrm{n} d s=6 V$

## PART B

Module I
Answer any two questions, each carries 5 marks.
Determine whether the alternating series $\sum_{k=1}^{\infty}(-1)^{k+1} \frac{k+7}{k(k+4)}$ is absolutely convergent
Find the Taylor series expansion of $f(x)=\frac{1}{x+2}$ about $x=1$
Find the interval of convergence and radius of convergence of $\sum_{k=1}^{\infty}(-1)^{k+1} \frac{(x+1)^{k}}{k}$

## Module II

## Answer any two questions, each carries 5 marks.

Find the local linear approximation L to the function $f(x, y, z)=x y z$ at the pointP $(1,2,3)$. Also compare the error in approximating $f$ by L at the point Q (1.001, 2.002, 3.003) with the distance PQ.
Locate all relative extrema and saddle points of $f(x, y)=2 x y-x^{3}-y^{2}$
If $u=f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x \frac{\partial u}{\partial x}+y \frac{\partial u}{\partial y}+z \frac{\partial u}{\partial z}=0$ $e^{-t} \boldsymbol{j}+t^{4} \boldsymbol{k}$ at $t=2$
A particle moves along the curve $\boldsymbol{r}=\left(t^{3}-4 t\right) \boldsymbol{i}+\left(t^{2}+4 t\right) j+$ $\left(8 t^{2}-3 t^{3}\right) \boldsymbol{k}$ where $t$ denotes time. Find
(i) the scalar tangential and normal components of acceleration at time $t=2$
(ii) the vector tangential and normal components of acceleration at time $t=2$

Find the equation to the tangent plane and parametric equations of the normal line to the ellipsoid $x^{2}+y^{2}+4 z^{2}=12$ a $t$ the point $(2,2,1)$

Module IV

## Answer any two questions, each carries 5 marks.

Reverse the order of integration and evaluate $\int_{0}^{1} \int_{x}^{1} \frac{x}{x^{2}+y^{2}} d y d x$
If R is the region bounded by the parabolas $y=x^{2}$ and $y^{2}=x$ in the first quadrant, evaluate $\iint_{R}(x+y) d A$
Use triple integral to find the volume of the solid bounded by the surface $y=x^{2}$ and the planes $y+z=4, z=0$.

## Module V

Answer any three questions, each carries 5 marks.
If $\boldsymbol{r}=x \boldsymbol{i}+y \boldsymbol{j}+z \boldsymbol{k}$ and $r=\|\boldsymbol{r}\|$, show that $\nabla \log r=\frac{\boldsymbol{r}}{r^{2}}$
Examine whether $\boldsymbol{F}=\left(x^{2}-y z\right) \boldsymbol{i}+\left(y^{2}-z x\right) \boldsymbol{j}+\left(z^{2}-x y\right) \boldsymbol{k}$ is a conservative
field. If so, find the potential function
Show that $\nabla^{2} f(r)=2 \frac{f^{\prime}(r)}{r}+f^{\prime \prime}(r)$, where $\boldsymbol{r}=x \boldsymbol{i}+y \boldsymbol{j}+z \boldsymbol{k}, r=\|\boldsymbol{r}\|$
Compute the line integral $\int_{c}\left(y^{2} d x-x^{2} d y\right)$ along the triangle whose vertices are
$(1,0),(0,1)$ and $(-1,0)$
Show that the line integral $\int_{c}(y \sin x d x-\cos x d y)$ is independent of the path and hence evaluate it from $(0,1)$ and $(\pi,-1)$

## Module VI

## Answer any three questions, each carries 5 marks.

Using Green's theorem, find the work done by the force field $\vec{f}(x, y)=$ $\left(e^{x}-y^{3}\right) \vec{\imath}+\left(\cos y+x^{3}\right) \vec{\jmath} \quad$ on a particle that travels once around the unit circle $x^{2}+y^{2}=1$ in the counter clockwise direction.
Using Green's theorem evaluate $\int_{c}\left(x y+y^{2}\right) d x+x^{2} d y$, where $c$ is the boundary of the area common to the curve $y=x^{2}$ and $y=x$
Evaluate the surface integral $\iint_{S} x z d s$, where S is the part of the plane
$x+y+z=1$ that lies in the first octant
Using divergence theorem, evaluate $\iint_{S} \mathrm{~F} . \mathrm{n} d s$ where
$\boldsymbol{F}=\left(x^{2}+y\right) \boldsymbol{i}+z^{2} \boldsymbol{j}+\left(e^{y}-z\right) \boldsymbol{k}$ and S is the surface of the rectangular solid bounded by the co ordinate planes and the planes $\mathrm{x}=3, \mathrm{y}=1, \mathrm{z}=3$
Apply Stokes's theorem to evaluate $\int_{c} F . d r$, where $\boldsymbol{F}=\left(x^{2}-y^{2}\right) \boldsymbol{i}+2 x y \boldsymbol{j}$ and c is the rectangle in the xy plane bounded by the lines $x=0, y=0, x=a$ and $y=b$

